Attachment A

Transmission–reflection analysis for localization of temporally successive multipoint perturbations in a distributed fiber-optic loss sensor based on Rayleigh backscattering

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A novel method is presented for the localization of multipoint loss-inducing perturbations in a distributed fiber-optic sensor. The proposed simple technique is based on measurement of the transmitted and the Rayleigh-backscattered powers of an unmodulated light launched into a sensing fiber. The positions of consecutive perturbations are determined by measuring the slopes of the dependence of normalized Rayleigh-backscattering power versus the square of normalized transmitted power. It is shown that these slopes uniquely depend on the positions of the disturbances along the test fiber. The method allows localization of any number of the perturbations that appear one after another at different positions along the test fiber without ambiguity. Good agreement is obtained between calculated and experimentally measured slopes for a loss that was consecutively induced near the source and remote ends of 2.844-km-long fiber. © 2003 Optical Society of America

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1. Introduction

The highest state of the art in optical sensing is achieved with optical fiber distributed sensors that allow measurement of a desired parameter along the test fiber. The regions where perturbations occur are usually localized by means of optical time-domain reflectometry (OTDR) or optical frequency-domain reflectometry.[1–4] All these methods utilize time- or frequency-modulated light sources that allow us to localize simultaneously a number of perturbations along the test fiber. Meanwhile, for some applications, it is important to detect and localize a rare but hazardous alarm condition that typically occurs as a single infrequent event, such as a pipe leak, fire, or explosion.

For such applications recently we proposed a novel simple and inexpensive measurement technique based on so-called transmission–reflection analysis (TRA). The TRA technique utilizes an unmodulated light source, power detectors, and a sensing fiber. A novel principle of the localization of loss-induced perturbation, based on measurement of transmitted and reflected[5] or Rayleigh-backscattered[6] powers, was used in TRA-distributed sensors. Localization of a strong disturbance was demonstrated with a maximum localization error of a few meters along a single-mode sensing fiber a few kilometers long.

Using the algorithm from earlier research,[5,6] we can localize only a single perturbation, which may appear along the sensing fiber. In practice the number of perturbations may affect the sensing fiber accidentally or from alarmlike disturbances. Usually, however, the probability of alarm conditions being detected is low, and therefore they will not appear at the same time.

In this paper we present theoretical and experimental evidence that the transmission–reflection analysis can be modified for detection and localization of a number of perturbations that appear one after another at different positions along the test fiber.

In Section 2 we show that localization of a single perturbation can be performed with a modified TRA method by measuring the slope of dependence of normalized backscattering power versus the square of normalized transmitted power. In Section 3 we present mathematical proof that localization of any number of consecutive perturbations can be done

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with a modified TRA method by measuring the corresponding slopes. An experimental verification of the proposed technique also is presented.

2. Single-Perturbation Localization

First, let us consider modification of the TRA method for localization of a single perturbation by using the slope of dependence of normalized Rayleigh-backscattering power on the square of normalized transmitted power. Generally, the TRA method is based on the unique relationships between normalized transmitted and Rayleigh-backscattered powers for different locations of the loss-induced disturbance along the sensing fiber.[6] Indeed equal losses equally decrease the transmitted power for any location of disturbance, but the value of Rayleigh backscattered power strongly depends on the disturbance location.

Let us consider the configuration with two plain fiber sections separated by a short fiber piece affected by a monitored condition (see Fig. 1). Plain fiber sections possess Rayleigh scattering and attenuation due to light absorption. The power reflection coefficient of each Rayleigh-scattering fiber segment can be calculated as[7,8]

$$R_{\delta L} = S(\alpha_s/2\alpha)[1 - \exp(-2\alpha\delta L)], \tag{1}$$

where α_s is the attenuation coefficient due to Rayleigh scattering, α is the total attenuation coefficient of the test fiber, δL is the length of the fiber segment, and recapture factor S for the fiber is defined as[9]

$$S = b(n_1^2 - n_2^2)/n_1^2, \tag{2}$$

where b depends on the waveguide property of the fiber and is usually in the range of 0.21–0.24 for the single-mode step-index fiber[9] and n_1 and n_2 are the refractive indices of the fiber core and cladding, respectively.

The short fiber piece is affected by monitored conditions that introduce additional light losses. The transmission of a short fiber piece is $t_1 \leq 1$. Let us assume that the scattering is relatively weak and a portion of the scattered light is very small. This allows us to simplify the analysis, neglecting multiple scattering in both directions. The reflections with coefficients r_1 and r_2 from the fiber source and remote ends, respectively, must be taken into account because even a weak reflection can be comparable with the backscattering. However, we can assume that $r_{1,2} \ll 1$ and neglect multiple reflections as well. In this case the relationship between the normalized transmitted T_{norm} and Rayleigh-backscattered R_{norm} powers for single perturbation can be expressed as[6]

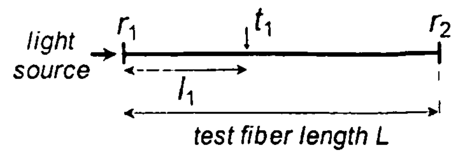

Fig. 1. Test fiber configuration for single perturbation: t_1, transmission of the loss-inducing segment; r_1, r_2, reflections from source and remote ends.

$$T_{\text{norm}}^2 = \frac{(S_\alpha + r_1)(R_{\text{norm}} - 1) - R_{\text{norm}}(S_\alpha - r_2)\exp(-2\alpha L) + S_\alpha \exp(-2\alpha l_1)}{S_\alpha[\exp(-2\alpha l_1) - \exp(-2\alpha L)] + r_2 \exp(-2\alpha L)}, \tag{3}$$

where L is the total sensing fiber length, l_1 is the distance from the source end to the disturbance location, and $S_\alpha = S(\alpha_s/2\alpha)$. The normalized transmitted T_{norm} and backscattered R_{norm} coefficients in Eq. (3) are defined as

$$T_{\text{norm}} = \frac{T}{T_{\max}} = t_1, \tag{4}$$

$$R_{\text{norm}} = \frac{R}{R_{\max}} = \frac{S_\alpha + r_1 - (S_\alpha - r_2)t_1^2 \exp(-2\alpha L) - S_\alpha(1 - t_1^2)\exp(-2\alpha l_1)}{S_\alpha + r_1 - (S_\alpha - r_2)\exp(-2\alpha L)}, \tag{5}$$

where $T_{\max}$ is the maximum transmittance of the initially undisturbed sensing fiber when $t_1 = 1$,

$$T_{\max} = \exp(-\alpha L), \tag{6}$$

and $R_{\max}$ is the maximum backscattering coefficients of the undisturbed optical system:

$$R_{\max} = S_\alpha + r_1 - (S_\alpha - r_2)\exp(-2\alpha L). \tag{7}$$

The slope of dependence of normalized backscattering power R_{norm} versus the square of normalizedtransmitted power T_{norm}^2 can be found from Eq. (3) as

$$\frac{\partial R_{\text{norm}}}{\partial (T_{\text{norm}}^2)} = \frac{S_\alpha[\exp(-2\alpha l_1) - \exp(-2\alpha L)] + r_2 \exp(-2\alpha L)}{S_\alpha + r_1 - (S_\alpha - r_2)\exp(-2\alpha L)}. \tag{8}$$

As we can see this slope uniquely depends on perturbation location l_1. Therefore the location of the single perturbation can be found from the experimentally measured slope as

$$l_1 = \frac{1}{2\alpha} \ln \left\{ \frac{S_\alpha}{(S_\alpha + r_1) \dfrac{\partial R}{\partial (T_{\text{norm}}{}^2)} + (r_2 - S_\alpha)\exp(-2\alpha L) \left[\dfrac{\partial R}{\partial (T_{\text{norm}}{}^2)} - 1 \right]} \right\}. \tag{9}$$

The relationship between normalized Rayleigh-backscattered power R_{norm} and the square of normalized transmitted power $T_{\text{norm}}{}^2$ is almost linear for a single perturbation that affects the test fiber in any location [see Eq. (3)]. Figure 2 shows the result of the numerical calculation of these relationships when additional losses occur at distances $l_{1,n} = n\Delta L$ from the source end of the test fiber, where $n = 0, 1, \ldots, 10$, and the interval between bending locations $\Delta L = 284.4$ m. Transmitted and backscattered powers were normalized with respect to their initial undisturbed values. A typical value for $b = 1/4.55$ for single-mode fiber[10] was used in the calculations.

The schematic diagram of a TRA-based fiber-optic sensor is shown in Fig. 3. Continuous-wave light emitted by an amplified spontaneous emission (ASE) optical fiber source operating near a 1550-nm wavelength with a linewidth of a few nanometers was launched into a standard single-mode step-index fiber 2.844 km long through a 3-dB coupler. The launched optical power was ~1.1 mW, and the attenuation coefficient of the test fiber α, which was measured with an OTDR, was equal to 0.19 dB/km. The optical isolator was used to cancel the influence of backreflections on the ASE source. An immersion of all fiber ends was employed to reduce backreflections. Standard power detectors were used to measure the transmitted and the Rayleigh-backscattered powers. To take into account the output laser-power fluctuations, a source power meter was utilized. To induce the bending losses in the sensing fiber, we used bending transducers, also shown schematically in Fig. 3. By tuning the bending transducer, we changed the normalized transmitted power from its initial undisturbed value equal to 1 to more than −30 dB. The bending losses were induced near the source end and the near remote end of the test fiber. Good agreement between experimental data and theory was obtained for $(\alpha_s/\alpha) = 0.68$, which is quite reasonable for the fiber with total attenuation coefficient $\alpha = 0.19$ dB/km.[10] Reflections from the source end and the remote end of the sensing fiber, which are equal to 4.7×10^{-6} and 1.5×10^{-5}, respectively, in our experiment, were also taken into account in the calculations.

Experimentally measured slopes for the bending losses, which were induced near remote and source ends of the test fiber, are equal to 0.109, and 3.63, respectively. These values agree well with the values calculated with Eq. (8), which are equal to 0.108 and 3.631, respectively.

The accuracy of the localization of excess loss with the TRA method strongly depends on the value of the induced loss. With the TRA method it is easier to localize a strong perturbation, but the localization of a weak perturbation requires higher accuracy in measurements of the transmitted and Rayleigh-backscattered powers. In contrast to this the accuracy of the localization of loss with the standard OTDR technique depends mainly on the duration of the optical test pulse and is practically independent of the value of loss. Figure 4 shows the relative localization error versus the induced excess loss for the TRA method calculated by using the data presented in Fig. 2. The relative localization error was

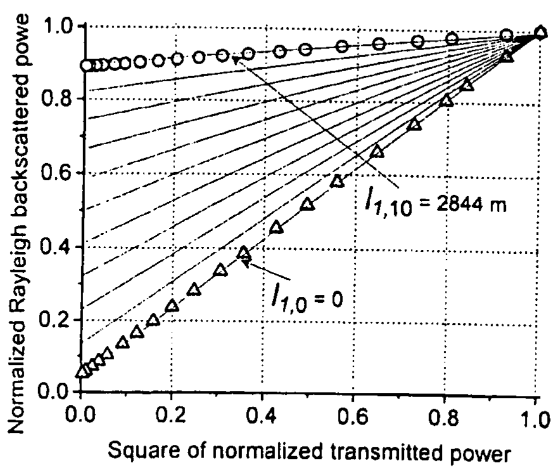

Fig. 2. Relations between normalized Rayleigh-backscattered power and the square of normalized transmitted power when additional losses occur at distances, $l_{1,n} = n\Delta L$, from the source end of the test fiber, where $n = 0, 1, \ldots, 10$ and the interval between bending locations, $\Delta L = 284.4$ m: ○, △, experimental results; solid lines, theoretical dependencies.

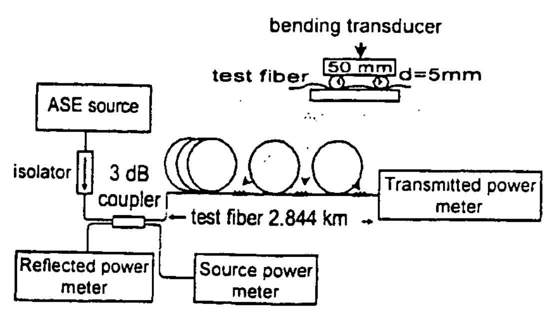

Fig. 3. Schematic diagram of the TRA fiber-optic sensor.

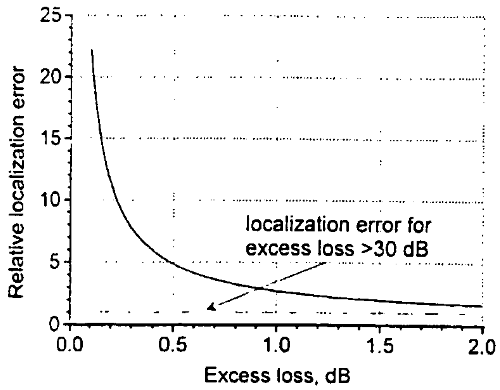

Fig. 4. Relative localization error versus excess loss for the TRA method.

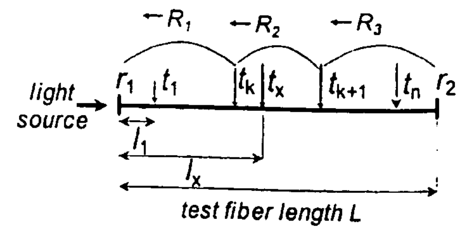

Fig. 5. Test fiber configuration for multipoint perturbations: $t_1 + t_n$, transmission of initially disturbed loss-inducing segments; t_x, transmission of the currently disturbed segment; r_1, r_2, reflections from source and remote ends.

determined as the localization error divided by their value for the strong induced loss that decreased the fiber transmission by more than 30 dB. As we can see the localization error for weak loss significantly exceeds the localization error for strong loss. In the experimental measurements in our previous experiment[11] the localization errors for the strong perturbation were equal to ± 1 m for the source and remote ends of a 2.844-km fiber. Therefore localization errors for 0.1- and 1-dB induced loss are equal to ± 22 and ± 3 m, respectively.

The measurement range and the maximum allowable loss for the TRA and the OTDR methods are likely to be the same, because both methods measure the backscattering power and the maximum range for both methods is namely restricted by the attenuation of test fiber.

Summarizing this section, we conclude that the modified TRA method allows us to localize the single perturbation by measuring the slope of dependence of the normalized Rayleigh-backscattered power on the square of the normalized transmitted power.

3. Localization of Multipoint Perturbations

Let us now verify that any number of consecutive perturbations can be localized with the modified TRA method. The proof is done by mathematical induction. In our analysis we consider the test fiber with the same properties and parameters as in the previous case. But now we use a configuration with a number of plain Rayleigh-scattering fiber sections separated by a number of short loss-inducing fiber pieces with transmissions $t_i \le 1$ (see Fig. 5). Let us assume that, according to the principle of mathematical induction, we have already determined the values and locations of the first n perturbations and demonstrated that we can find the position of the next $(n + 1)$th perturbation without ambiguity. Here we consider only the perturbations that appear one after another at different positions along the test fiber. So, at this moment all initial n perturbations induce fixed known losses at known locations and only a new $(n + 1)$th perturbation can modify the reflectivity and transmittance of the test fiber.

Because we know the positions and values of all n initial perturbations, we can number these according to their positions along the test fiber (see Fig. 5). We also suppose without loss of generality that a new perturbation is located at a distance l_x between the kth and $(k + 1)$th initial perturbations, where k is unknown. Assuming that the scattering and reflections from the fiber ends are relatively weak, we neglect multiple scattering and reflections in both directions. In this case the derivative of the normalized backscattering power with respect to the square of the normalized transmitted power can be expressed as (see Appendix A)

$$\frac{\partial R_{\text{norm}}}{\partial (T_{\text{norm}}{}^2)} = \frac{1}{R_{\max} \prod_{j=0}^{n} t_j{}^2} \Bigg\{ S_\alpha \prod_{j=0}^{k} t_j{}^2 [\exp(-2\alpha l_x) - \exp(-2\alpha l_{k+1})] + S_\alpha \sum_{i=k+1}^{n} [\exp(-2\alpha l_i) - \exp(-2\alpha l_{i-1})] \prod_{j=0}^{i} t_j{}^2 + T_L{}^2 r_2 \prod_{j=0}^{n} t_j{}^2 \Bigg\}, \tag{10}$$

where $T_L = \exp(-\alpha L)$ is the transmission coefficient of the test fiber with length L and l_i are the location of the initial loss-inducing short segments (see Fig. 5).

This derivative (or the slope of the dependence of normalized backscattering power versus the square of the normalized transmitted power) depends only on the backscattering from the plain segments located on the right side of the unknown perturbation. The plain segments located on the left of the unknown perturbation do not affect the slope.

Let us induce for $0 \le k \le n + 1$ a helper function $F(k, n)$:

$$F(k, n) = \frac{1}{R_{\max} \prod_{j=0}^{n} t_j{}^2} \Bigg\{ S_\alpha \sum_{i=k}^{n} [\exp(-2\alpha l_i) - \exp(-2\alpha l_{i-1})] \prod_{j=0}^{i} t_j{}^2 + T_L{}^2 r_2 \prod_{j=0}^{n} t_j{}^2 \Bigg\}. \tag{11}$$

The helper function has a structure similar to the expression for the slope [see Eq. (10)], and it de-

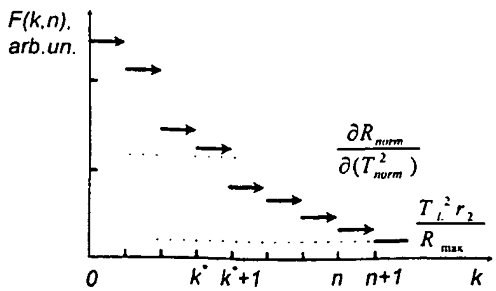

Fig. 6. Preliminary localization of the $(n + 1)$th perturbation with the helper function $F(k, n)$.

creases with k for any n (see Fig. 6). The contribution in the slope due to backscattering from segment $[l_x, l_{k+1}]$, which is associated with the term $[\exp(-2\alpha l_x) - \exp(-2\alpha l_{k+1})]$ in Eq. (10), is less than the possible contribution owing to backscattering from the full segment $[l_k, l_{k+1}]$, which is associated with the term $[\exp(-2\alpha l_k) - \exp(-2\alpha l_{k+1})]$ in Eq. (11). Comparing Eqs. (10) and (11), we conclude that, if the measured value of the slope for unknown perturbation satisfies

$$F(k^* + 1, n) < \frac{\partial R_{\mathrm{norm}}}{\partial (T_{\mathrm{norm}}{}^2)} < F(k^*, n), \qquad (12)$$

the unknown perturbation is located between the k^*th and $(k^* + 1)$th initial perturbations (see Fig. 6). Note that, if the measured slope of the unknown perturbation is equal to $F(0, n)$, the new perturbation affects the testing fiber near the source end of the test fiber. If the slope is equal to $F(n + 1, n) = \exp(-2\alpha L)r_2/R_{\max}$, the unknown disturbance is located near the remote end of the test fiber.

Finally, the exact location of the sought-for short loss segment can be found:

$$l_x = \frac{1}{2\alpha} \ln \left\{ \frac{S_\alpha}{\left[\dfrac{\partial R_{\mathrm{norm}}}{\partial (T_{\mathrm{norm}}{}^2)} - F(k^* + 1, n) \right] R_{\max} \prod_{j=k+1}^{n} t_j^2 + S_\alpha \exp(-2\alpha l_{k^*+1})} \right\}. \qquad (13)$$

In Section 2 we presented a method for single-perturbation localization with a modified TRA technique. Now we have demonstrated the algorithm for localization of a new perturbation when the values and locations of all initial n perturbations are known. Therefore, according to the principle of mathematical induction, we have demonstrated that a modified TRA method can be implemented for the localization of any number of consecutive perturbations.

Figure 7 shows the experimental dependencies of normalized Rayleigh-backscattered powers versus the square of normalized transmitted powers for the bending losses consequently induced near the remote and source ends of the test fiber. Measurements were performed as follows. Initially the perturbation occurred near the remote end of the test fiber.

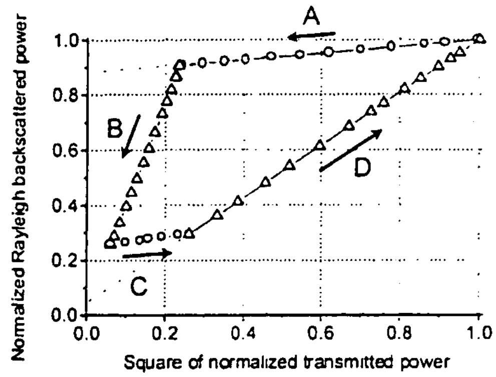

Fig. 7. Relations between normalized Rayleigh-backscattered power and the square of the normalized transmitted power for the bending losses consequently induced near the remote and source ends of the test fiber.

The increase in the losses leads to a decrease in transmitted power (line A in Fig. 7). When the square of the normalized transmittance decreases to a value equal to 0.241 of its initial undisturbed magnitude, we stop to increase the bending losses. Afterward, keeping constant losses near the remote end, we induce additional losses near the source end of the test fiber. This loading continues until the value of the square of normalized transmittance decreases to 0.061 (line B in Fig. 7). Then, keeping the same value of losses near the source end, we gradually remove the losses near the remote end of the test fiber (line C in Fig. 7). Finally, when the losses near the source end are eliminated, all the parameters return to their initial undisturbed values (line D).

All the experimental dependencies presented in Fig. 7 possess linear behavior. Experimental data show good agreement with theoretical prediction. Namely, experimental dependencies A and D practically coincide with the calculated dependencies when Eq. (3) is used (see Figs. 2 and 7). Experimentally measured slopes for lines A, B, C, and D, equal to 0.109, 3.63, 0.109, and 0.955, respectively, agree with values calculated from using Eqs. (8) and (10), equal to 0.108, 3.631, 0.108, and 0.957, respectively. Note that the slopes were the same for loading and unloading dependencies. The localization errors that were estimated from the difference between the measured and the calculated slopes do not exceed 2 m for any location of perturbation. Note, however, that in practice different noise origins and system imperfections such as temporal drifts of fiber and photodetectors parameters, additional uncontrolled losses, etc. may also contribute to a decrease in accuracy.

The TRA method can be implemented for localiza-

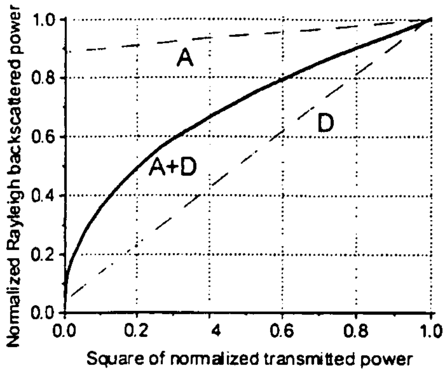

Fig. 8. Relations between normalized Rayleigh-backscattered power and the square of the normalized transmitted power for two perturbations, A + D, synchronously and, A and D, independently induced near the remote and source ends of the test fiber.

tion of any number of consecutive perturbations that occur one after another along the test fiber during the monitoring period. In contrast to the OTDR the proposed method cannot be used for localization of the perturbations in a fiber optical system already installed and that already has many induced losses. The other natural question to be addressed about the TRA method is the following: What happens when two perturbations affect the test fiber simultaneously? To answer this question, consider calculating the dependence of normalized Rayleigh-backscattered power versus the square of the normalized transmitted power for two equal perturbations that induce losses near the source and remote ends at the same time (curve A + D in Fig. 8). The dependence exhibits clear nonlinear behavior. As shown above for any number of consecutive perturbations this dependence should be linear. Figure 8 also shows the normalized Rayleigh-backscattered power versus the square of the normalized transmitted power for perturbations that affect the testing fiber one after another near the remote (line A) and source (line D) ends. Both of the latter dependencies exhibit clear linear behavior. The nonlinear behavior of the dependencies of normalized Rayleigh backscattered power versus the square of normalized transmitted power indicates that the testing fiber is affected by two or more perturbations simultaneously. When the particular dependence shown in Fig. 8 (curve A + D) is used, it is possible to localize at least one perturbation. Indeed the value of the normalized Rayleigh backscattered power at the point at which normalized transmitted power is equal to zero directly shows the location nearest to the source-end perturbation. Therefore, analyzing the curve A + D, we conclude that two perturbations affect the test fiber simultaneously and one of the perturbations induces losses near the source end. Nevertheless a complete analysis of the different possibilities, even for two synchronous events, is noticeably complicated and beyond the scope of this paper.

4. Summary

We have demonstrated that the modified TRA method can be implemented for the localization of any number of loss-inducing perturbations that appear one after another at different positions along the test fiber. The localization of the perturbations with a modified TRA method was performed by measuring the slope of dependence of the normalized Rayleigh-backscattered power versus the square of normalized transmitted power. It was shown that these slopes uniquely depend on the position of the perturbations along the test fiber. We believe that the proposed TRA technique should be attractive in the eventual realization of a compact and inexpensive distributed fiber-optic sensor.

Appendix A: Calculation of the Slope

Let us consider an optical system that consists of a number of Rayleigh-scattering plain segments separated by a number of short loss-inducing fiber pieces (see Fig. 5). The transmittance of initial n loss-inducing short segments located at distances l_j from the source end is $t_j \leq 1$, where $j = 1, n$. The transmittance of an unknown loss-inducing segment that is located at distance l_x is t_x. Neglecting multiple scattering and reflections in both directions, we can calculate the normalized power-reflection coefficient of the optical system as

$$R_{\mathrm{norm}} = \frac{R_1 + R_2 + R_3}{R_{\max}}, \tag{A1}$$

where R_1 corresponds to Rayleigh backscattering from segments placed on the left of the unknown perturbation plus the Fresnel reflection from the source end of the test fiber:

$$R_1 = r_1 + S_\alpha \sum_{i=0}^{k-1} \left(T_{l_i}^2\{1 - \exp[-2\alpha(l_{i+1} - l_i)]\} \prod_{j=0}^{i} t_j^2\right). \tag{A2}$$

R_2 is the Rayleigh-backscattering coefficient of two segments that are placed around the unknown perturbation:

$$R_2 = S_\alpha \prod_{j=0}^{k} t_j^2 (T_{l_k}^2\{1 - \exp[-2\alpha(l_x - l_k)]\} + T_{l_x}^2\{1 - \exp[-2\alpha(l_{k+1} - l_x)]\} t_x^2), \tag{A3}$$

and R_3 is the power-reflection coefficient for segments that are placed on the right of the disturbed region plus the term related to the reflection from the remote end of the test fiber:

$$R_3 = t_x^2 \left(S_\alpha \sum_{i=k-1}^{n} T_{l_i}^2\{1 - \exp[-2\alpha(l_{i+1} - l_i)]\} \times \prod_{j=0}^{i} t_j^2 + T_L^2 r_2 \prod_{j=0}^{n} t_j^2\right), \tag{A4}$$

where $T_{li} = \exp(-\alpha l_i)$ is the transmission coefficient of the fiber segment with length l_i (see Fig. 5) and r_1, r_2 are the reflections coefficients from the fiber source and remote ends, respectively. In Eqs. (A2)–(A4) we assign $l_0 \equiv 0$, $t_0 \equiv 1$, and $l_{n+1} \equiv L$. We emphasize that the condition $t_0 \equiv 1$ does not mean that perturbation cannot appear near the source end of the test fiber. The disturbance can affect the fiber near the source end, but it should be marked as the first perturbation with transmittance t_1 at the $l_1 = 0$ distance.

The normalized transmitted coefficient of the optical system, which is affected by all $n + 1$ perturbations, is defined as

$$T_{\text{norm}}^2 = t_x^2 \prod_{j=1}^{n} t_j^2. \tag{A5}$$

Note that changes in the first n perturbations are almost finished before a new one is started, so that t_j = constant for $j = 1, n$. Therefore the total normalized transmitted power for all $n + 1$ perturbations T_{norm}^2 can change only because of the change in current perturbation. The differential $\partial(T_{\text{norm}}^2)$ is

$$\partial(T_{\text{norm}}^2) = \partial(t_x^2) \prod_{j=1}^{n} t_j^2, \tag{A6}$$

The derivative of the normalized backscattering power with respect to the square of the normalized transmitted power is expressed as

$$\begin{aligned} \frac{\partial R_{\text{norm}}}{\partial(T_{\text{norm}}^2)} &= \frac{1}{R_{\max}} \left[\frac{\partial R_1}{\partial(T_{\text{norm}}^2)} + \frac{\partial R_2}{\partial(T_{\text{norm}}^2)} + \frac{\partial R_3}{\partial(T_{\text{norm}}^2)} \right] \\ &= \frac{1}{R_{\max} \prod_{j=0}^{n} t_j^2} \Bigg\{ S_\alpha \prod_{j=0}^{k} t_j^2 [\exp(-2\alpha l_x) \\ &\quad - \exp(-2\alpha l_{k+1})] + S_\alpha \sum_{i=k+1}^{n} [\exp(-2\alpha l_i) \\ &\quad - \exp(-2\alpha l_{i+1})] \prod_{j=0}^{i} t_j^2 + T_L^2 r_2 \prod_{j=0}^{n} t_j^2 \Bigg\}, \end{aligned} \tag{A7}$$

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